SYNCHRONOUS BEHAVIOR IN SMALL POPULATIONS OF LIGHT-CONTROLLED OSCILLATORS

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[*] Partially supported by Belgian Technical Cooperation (BTC) and the European Project Leurre (IST-2001-35506)

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Abstract— *We describe the electronic implementation of light-controlled oscillators (LCOs) and the synchronous behavior that is observed when two or more oscillators are in interaction. Numerical results are presented for a small group of oscillators in a ring configuration and for globally coupled LCOs.*

I. INTRODUCTION

Synchronization constitutes a common phenomenon that is present both in natural systems and artificial devices. It entails the interaction of two or more self-sustained oscillators and it consists in an adjustment of rhythms of these oscillating objects due to their weak interaction [1]. Since the seminal works on the synchronization property in a triode generator performed by Eccles, Vincent, van der Pol and Appleton [1], electric and electronic components have been widely used to study synchronization (see e.g. [2], [3], [4]) and other nonlinear phenomena (see e.g. [5], [6], [7]). Most of these works deal with sinusoidal-like oscillators that allow to perform analysis using the same mathematical approaches found in classical works (see e.g. [8], [9]). Among the numerous examples where synchronization is observed in nature, one of the most astonishing is the flash synchronization in certain firefly species [10]. The synchronous flashing behavior can be understood as a self-organizing process [11]. Our work deals with the synchronous behavior of relaxation oscillators controlled by light pulses, which mimic the firefly flashing behavior. In §II we detail the electronics of an LCO, and in §III we present a model describing the LCOs' behavior and some results for two interacting oscillators. Finally, in §IV we examine the situation for small populations of LCOs in a ring configuration and for globally coupled oscillators.

II. LCO DESCRIPTION AND MODEL

An LCO is a device made up of two RC circuits, a timer chip LM555 in its astable functioning mode, infrared LEDs and photo-sensors [12], [13], [14] (Fig. 1). The optoelectronic components allow the LCOs to interact with one another. Basically, an LCO is a relaxation oscillator in the sense that it has two time scales characterized by the binary variable $\epsilon(t)$: within each cycle there are intervals of slow (charging stage, $\epsilon(t) = 1$) and fast (discharging stage, $\epsilon(t) = 0$) motion. The form of the oscillation is very different from a sinusoidal wave. The period is determined by the two external RC circuits and the output waveform takes the form of a pulse signal with minimum and maximum values set at $\frac{V_M}{3}$ and $\frac{2V_M}{3}$ respectively, V_M being the value of the supply voltage. These threshold voltages determine the value of $\epsilon(t)$:

$$\begin{array}{ll} \textbf{If } V(t) = \frac{V_M}{3} \ \wedge \ \epsilon(t) = 0 & \Rightarrow \ \epsilon(t_+) = 1. \\ \textbf{If } V(t) = \frac{2V_M}{3} \ \wedge \ \epsilon(t) = 1 & \Rightarrow \ \epsilon(t_+) = 0. \end{array} \tag{1}$$

Resistors R_λ and R_γ can be modified manually and are used to set the two time delays constituting the period. The period is the sum of the charging stage related to the time constant $\frac{1}{\lambda} = (R_\lambda + R_\gamma)C$, and the discharging stage of the same capacitor that is related to the time constant $\frac{1}{\gamma} = R_\gamma C$. LEDs wired to the output of the LM555 chip emit a light flash during the discharging stage of the period. The light beam is directed to the photo-sensors of the neighboring LCOs, establishing an optical coupling characterized by the variable β. Depending on the phase difference between interacting LCOs, a flash causes a neighbor to shorten its charging stage and/or to lengthen its discharging one. In our LCOs, resistors take the following values:

$$\begin{aligned} R_\lambda &= 68\,\mathrm{k}\Omega \,+\, [0, 50]\,\mathrm{k}\Omega \\ R_\gamma &= 1.2\,\mathrm{k}\Omega \,+\, [0.0, 1.0]\,\mathrm{k}\Omega, \end{aligned}$$

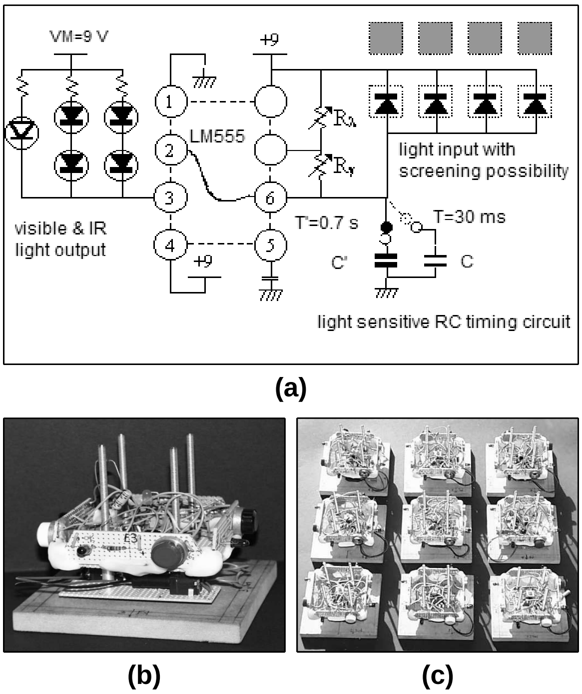

Fig. 1. (a) Block diagram of the LCO. (b) View of a single LCO. (c) Group of nine LCOs.

being the numbers between brackets, the interval value that can be added to the resistors, and the value that we use for the capacitor in order to perform measurements with an oscilloscope is $C = 0.47\ \mu$F which produces a period $T \sim$ 30 ms.

If we consider a system composed of N LCOs, the equation we use to model the voltage evolution for the i^{th} LCO is:

$$\begin{aligned}\frac{\mathrm{d}V_i(t)}{\mathrm{d}t} &= \underbrace{\lambda_i(V_{Mi} - V_i(t))\epsilon_i(t)}_{\text{charging term}} \\ &\quad - \underbrace{\gamma_i V_i(t)[1-\epsilon_i(t)]}_{\text{discharging term}} \\ &\quad + \underbrace{\sum_{j=1}^{N} \beta_{ij}\delta_{ij}[1-\epsilon_j(t)]}_{\text{interaction term}}, \qquad (2)\end{aligned}$$

where

$$\delta_{ij} = \begin{cases} 1\,, & \text{if } i \neq j \text{ and they may interact} \\ 0\,, & \text{otherwise} \end{cases}$$

indicates whether or not LCOs i and j interact.

III. Synchronization in Two LCOs

Using (2) we have been able to reproduce experimental results in 3 configurations, firstly with two LCOs in a master–slave situation, secondly for mutual interaction and finally for three LCOs in a line configuration. The phase response curve (PRC) was obtained and an analysis of two identical LCOs was performed, showing that the system tends always towards synchronization except for a specific initial condition for which the interacting LCOs are in an unstable anti-synchronous stationary state [14].

When performing experimental measurements, we always have non- identical LCOs (the electronic components values are not the same and are affected by random errors, there are always some small perturbations that cannot be controlled, and so on). In order to study higher order synchronization for 2 LCOs, we fixed the period of LCO_1 and modified systematically the period of our reference oscillator LCO_2 (period mismatch). Under these conditions, we define $n : m$ synchronization (n pulses of LCO_2 within m oscillatory cycles of LCO_1) as the regime in which *phase-locking* occurs:

$$\mid n\phi_1 - m\phi_2 \mid < k. \qquad (3)$$

Here, k is a constant that guarantees a bounded phase difference, which is equivalent to the condition of *frequency-locking* [1] that can be expressed in terms of the LCOs' periods as:

$$\langle T_1 \rangle = \frac{n}{m}\langle T_{2(ref)} \rangle, \qquad (4)$$

where brackets mean time averaging. It is important to note that in the case in which the LCO firings are almost simultaneous, k must be of the same order as the discharging time. Arnold-tongue structures are useful for the investigation of returned periodicities [15]. We have obtained from experimental and numerical results the Arnold tongues in the plane (period detuning, coupling strength) for two LCOs in interaction (Fig. 2). We can observe from Fig. 2 that there are two possible types of synchronization for the system. The first involves phase-locking and frequency-locking with the constraint that the period for both LCOs must not change in the synchronous regime, i.e. T_i =constant. The other possibility is that *modulation* can occur, i.e. the period of one of the oscillators varies from cycle to cycle and the firings are not equidistant [1]. We note that modulation appears for large values of the interaction strength.

IV. Synchronization in Groups of LCOs

From the analytical, numerical and experimental results, we see that LCOs' behavior is very sensitive to initial conditions in the sense that slight changes in

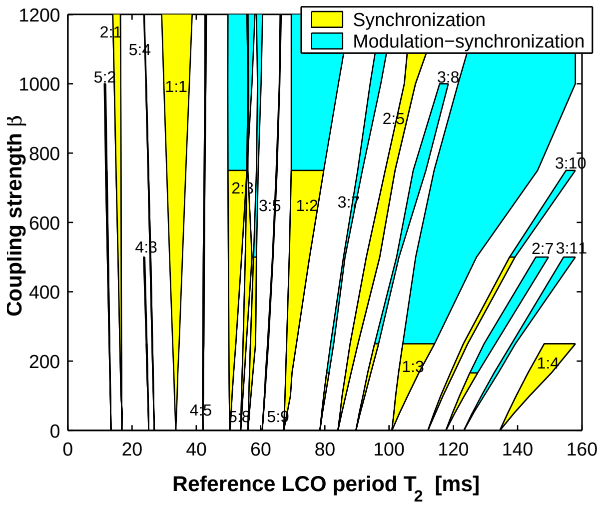

Fig. 2. Arnold tongues obtained for two interacting LCOs when $T_1 = 33.620$ ms is fixed and T_2 is mismatched.

the initial conditions can drive the system to different states or can lengthen or shorten the synchronous time [14]. In practice it is not possible to control the LCOs' initial conditions precisely, so that a statistical analysis is necessary in order to study the synchronous behavior of LCO populations. We have studied two types of LCO groups: a ring configuration and a global coupling between LCOs.

A. LCOs in a Ring Configuration

Here we suppose that the LCOs are identical and that they can interact with two neighbors, the distance between neighboring LCOs being constant, i.e. the coupling strength is the same for each pair of LCOs. We solved (2) numerically for rings of between 2 and 25 LCOs, varying the coupling strength from $\beta = 10$ to $\beta = 1000$ in steps of $\Delta\beta = 10$ (100 simulations for each case). The numerical results we obtained, allow us to construct the surface shown in Fig. 3(a) and its respective projection onto the β–N plane, where N is the number of LCOs. From Fig. 3(b), we note that global synchrony (almost simultaneous firings) is present over a broader range for weak coupling, except for 5 LCOs, for which the percentage of total synchronization events fell sharply. For greater coupling strength, there is a tendency for the number of successful total synchronization events to decrease as the number of LCOs increases. Nevertheless, we must point out that if we consider equality of periods and any bounded phase difference, we find situations in which certain stable patterns quickly arise in the system, as shown in Fig. 4, in which not all the LCOs flash at the same time. There are two groups of LCOs that flash almost simultaneously: (i) LCOs 1, 2 and 3

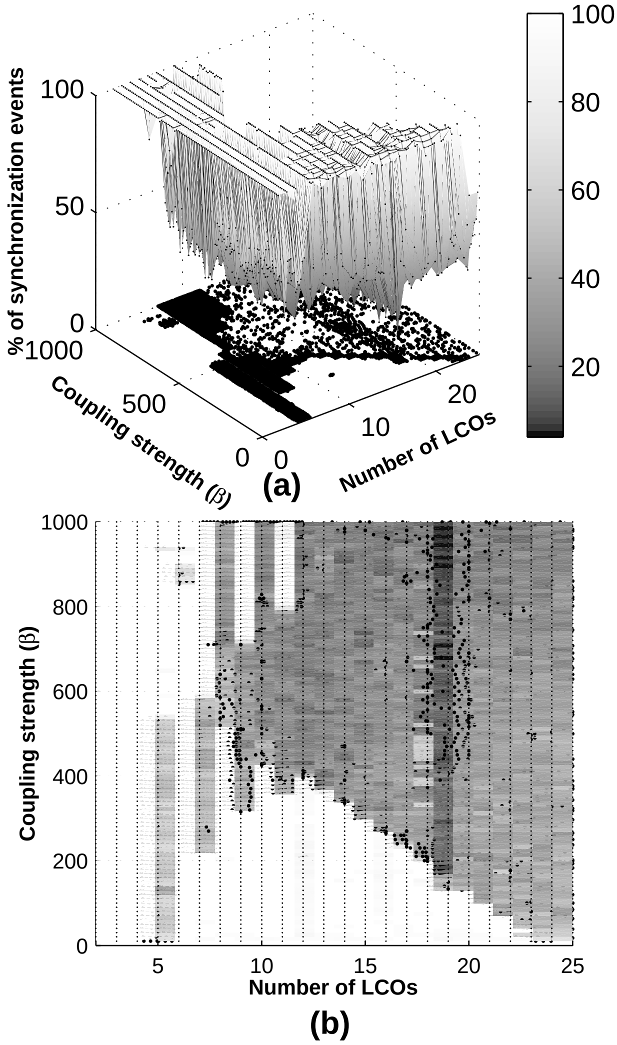

Fig. 3. (a) Surface generated with the percentage of total synchronized events as a function of coupling strength and the number of LCOs. (b) Projection onto the plane Coupling strength vs. Number of LCOs.

(despite the phase slip that occur between the phases of LCOs 1 and 3 respect to the phase of LCO_2) and (ii) LCOs 4 and 5.

B. Global Coupling

In order to study all-to-all coupling (each element interacts with all others) between LCOs, we considered a "square" arena consisting of 2500 locations (50 X 50 cells) each of which can be occupied by an LCO. We performed 100 simulations with random initial conditions and random LCO positions, with between 2 and 25 LCOs. Experimentally, the coupling strength (β) is found to depend on the distance (r) between the LCOs, as $\beta_{ij} \propto \frac{1}{r_{ij}^{\alpha}}$, where the exponent α was found to take the value 2.11. We considered approximately 15000 firing events for this study. Again, we must distinguish between synchronization with al-

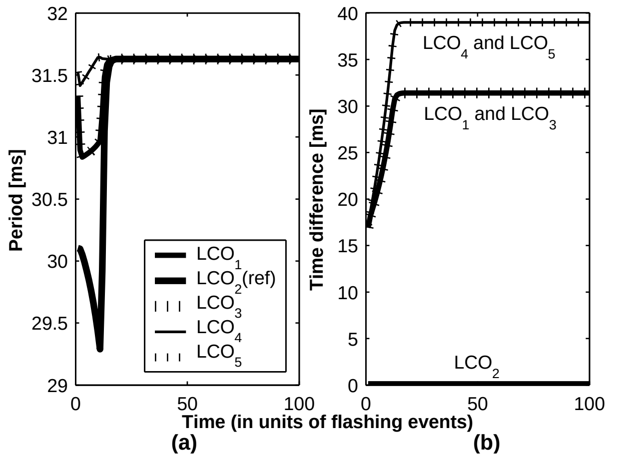

Fig. 4. Results for five LCOs in a ring configuration. (a) LCO's period evolution. (b) Time difference evolution between LCO's firings. Coupling strength $\beta = 280$.

most simultaneous firings and synchronization with phase-locking and frequency-locking. In Fig. 5(a) we have computed the number of simulations for which total synchronization is achieved (all the LCOs fire almost simultaneously). Moreover, Fig. 5(a) shows that the number of LCOs plays an important role for the synchronization phenomenon. Total synchronization tends to zero as the number of LCOs increases. Nevertheless, it is clear that the system contains several synchronized clusters (Fig. 5(b) and (c)) that can give rise to spatio-temporal patterns. Moreover, the density of LCOs could also be important for global synchrony.

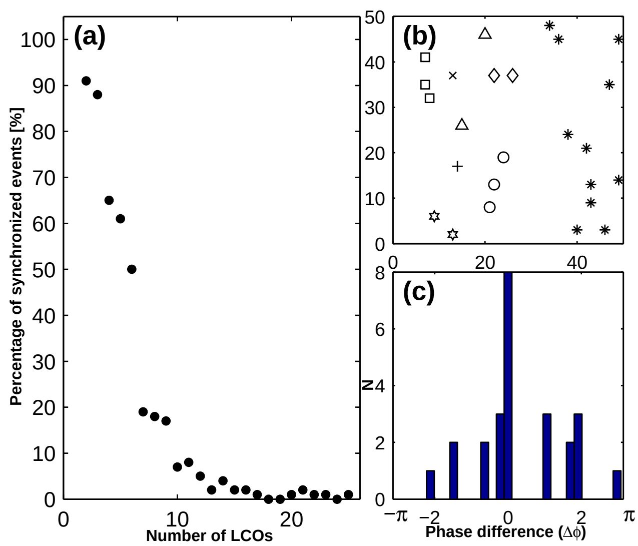

Fig. 5. (a) Percentage of total synchronization events as a function of the number of LCOs. (b) 25 LCOs randomly distributed in the arena and synchronous clusters (LCOs with same marker style) formed after approximately 15000 flashing events. (c) Phase difference distribution for the LCOs in (b).

V. Conclusions and Perspectives

Our work shows the richness that a simple electronic circuit can exhibit at the level of synchronous behavior. For a small number of LCOs, the system seems to have synchronization robustness, but when the number of LCOs increases, the system tends to form small synchronous clusters instead of achieving global synchrony. This type of realistic device could constitute a useful tool to understand synchronous behavior in some biological systems, especially in fireflies. On the other hand, the ability to synchronize exhibited by the LCOs could find some applications in collective robotics.

References

[1] A. Pikovsky, M. Rosenblum and J. Kurths. *Synchronization – A Universal Concept in Nonlinear Sciences*. (Cambridge University Press, Cambridge, 2001).

[2] I. Blekhman. *Synchronization in Science and Technology*. (Asme Press, New York, 1988).

[3] A. Kittel, J. Parisi and K. Pyragas. Generalized Synchronization of Chaos in Electronic Circuit Experiments. *Physica D* **112**, pp. 459–471, 1998.

[4] A. Dabrowski, Z. Galias and M. Ogorzalek. Observations of Phase Synchronization Phenomena in One-Dimensional Arrays of Coupled Chaotic Electronic Circuits. *Int. J. Bifurc. and Chaos* **10** pp. 2391–2398, 2000.

[5] L. O. Chua, C. W. Wu, A. Huang, and Zhong Guo-Qun. A Universal Circuit for Studying and Generating Chaos. I. Routes to chaos. *IEEE Trans. on Circ. and Sys. I* **40**, pp. 732–744, 1993.

[6] S. Tanaka, T. Matsumoto, and L. O. Chua. Bifurcation Scenario in a Driven R-L-Diode Circuit. *Physica D* **28**, pp. 317–344, 1987.

[7] D. V. Ramana Reddy, A. Sen, and G. L. Johnston. Experimental Evidence of Time-Delay-Induced Death in Coupled Limit Cycle Oscillators, *Phys. Rev. Lett.* **85**, pp. 3381–3384, 2000.

[8] Y. Kuramoto, I. Nishikawa. Statistical Macrodynamics of Large Dynamical Systems. Case of a Phase Transition in Oscillator communities. *J. Statist. Phys.* **49**, pp. 569–605, 1987.

[9] P. C. Matthews, and S. H. Strogatz. Phase Diagram for the Collective Behavior of Limit Cycle Oscillators. *Phys. Rev. Lett.* **65**, pp. 1701–1704, 1990.

[10] J. Buck, and E. Buck. Synchronous Fireflies. *Sci. Am.* **234**, pp. 74–85, 1976.

[11] S. Camazine, J. L. Deneubourg, S. Franks, J. Sneyd, G. Theraulaz, and E. Bonabeau. *Self-Organization in Biological Systems*. (Princeton University Press, Princeton, 2001).

[12] W. Garver, and F. Moss. Electronic Fireflies. *Sci. Am.* **269**(6), pp. 128–130, 1993.

[13] J. L. Guisset, J. L. Deneubourg, and G. M. Ramírez Ávila. The Phase Information Associated to Synchronized Electronic Fireflies. *arXiv: nlin.AO/0206036*, 2002.

[14] G. M. Ramírez-Ávila, J. L. Guisset, and J. L. Deneubourg. Synchronization in Light-Controlled Oscillators. *Submitted to Physica D*, 2002.

[15] R. Stoop, K. Schindler, and L. A. Bunimovich. When Pyramidal Neurons Lock, When They Respond Chaotically, and When They Like to Synchronize. *Neurosci. Res.* **36**, pp. 81–91, 2000.