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Synchronization conditions in two coupled pendulums

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Abstract

Based in a damped pendulum discrete model, we have studied the synchronization conditions for two coupled pendulums, varying both the pendulums features and the coupling conditions. We found the basin of attraction for several situations in which the control parameters were fixed. Varying the control parameters (length, mass and damping coefficient), we have found phase diagrams related to the initial conditions of one of the pendulums; on these diagrams we have identified synchronization regions. We emphasize the synchronization with a winding number $\rho \approx 1$ (synchronization 1:1); nevertheless, other synchronization orders are possible ($\rho \neq 1$).

Keywords: Synchronization, coupled oscillators.

Introduction

Synchronization phenomena is very common in nature [1-3], many systems exhibit this behavior. Historically, synchronization was discovered by Huygens observing two coupled pendulums. This simple system possesses a rich behavior when considering damping and coupling variations. The aim of our work is to study the synchronous behavior of the system when these features are varied as well as some other parameters such as the pendulums' length and mass. On the other hand, we work simultaneously with experiments in order to adapt our model to real situations. Moreover, the above allows us to compare the results obtained experimentally with those obtained numerically.

Model

We propose a minimal discrete model, which consider two coupled damped pendulums. This model was used before [4] and also the results obtained with it have been compared with experimental results exhibiting good agreement with them. The equations of the model can be written as:

$$\theta^{(1)}_{t+1} = \theta^{(1)}_{t} + b^{(1)}\left(\theta^{(1)}_{t} - \theta^{(1)}_{t-1} - K^{(1)}\sin\theta^{(1)}_{t}\right) + \frac{w}{m^{(1)}}\sin\theta^{(2)}_{t} \qquad (1)$$

$$\theta^{(2)}_{t+1} = \theta^{(2)}_{t} + b^{(2)}\left(\theta^{(2)}_{t} - \theta^{(2)}_{t-1} - K^{(2)}\sin\theta^{(2)}_{t}\right) + \frac{w}{m^{(2)}}\sin\theta^{(1)}_{t} \quad ,$$

where the superscripts 1 y 2 identify each of the coupled pendulums, and w are linked to the coupling strength and it depends on the distance among the pendulums, and the stiffness of the material used to couple both pendulums. We also consider the following relationships:

$$b^{(i)} = \frac{1}{1 + \frac{\lambda^{(i)}}{m^{(i)}}\Delta t} \qquad \text{y} \qquad K^{(i)} = \frac{g\Delta t^2}{l^{(i)}} \quad ,$$

(2)
with $i = 1, 2$.
Due to the great quantity of parameters which can vary the behavior of this system, we try to consider all the possible values of these parameters in order to characterize the condition of synchronization of these two pendulums.

Results

We use the relation between the period of the two pendulums to characterize the regions were synchronizations can occur, using the winding number $\rho = T_2/T_1 \approx 1$ what implies a synchronization 1:1. First we study the conditions of synchronization, varying the initial condition of oscillation of the two pendulums under the different characteristics of the two pendulums. We show these results for different lengths of the two pendulum, different values of mass, and also we change the damping coefficient and the coupled condition. In all these cases we can see that the regions of synchronization change for this system.

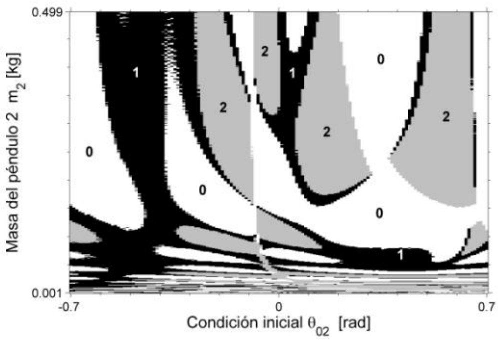

Figura 1. Phase diagram m_2 vs. θ_{02} showing the synchronous regions with a winding number. $\rho = 1.0000 \pm 0.0005$ (dark region identified with 1), when $l_1 = l_2 = 1.00$ m. Other regions can also be identified: $\rho = (0,1)$ y $\rho = (1,2)$, (regions identified with 0 and 2).

Using these results we can also analyze the synchronization regions for a long range of values of the parameters. First we study the importance of the length of the second pendulum, keeping fix the length of the first one. We do the same with the mass of the pendulums (Figure 1), with the coupling condition and also with the dissipation factor. This let us identify not only regions of synchronization 1:1 but also other orders of synchronization.

Conclusion

We show different conditions of synchronization for this system with the possibility to also identify quasi periodic and chaotic regions. The possibility to study

References

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