INFLUENCE OF UNIFORM NOISE ON TWO LIGHT-CONTROLLED OSCILLATORS

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October 28, 2006

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Short title: INFLUENCE OF UNIFORM NOISE ON TWO LCOS.

Abstract

We study the influence of uniform noise on a system of two light-controlled oscillators (LCOs) under three different configurations: uncoupled, master–slave and mutually coupled LCOs. We find that noise can induce desynchronization via a phase transition-like phenomenon depending on the noise intensity and the characteristics of the LCOs.

keywords: synchronization; coupled oscillators; optoelectronic devices

1 Introduction

Synchronization is a common phenomenon that occurs in oscillators coupled among themselves or driven by external time-dependent forcings. Recently, a new class of oscillators has been introduced which exhibits synchronization in a very simple experimental setup [Ramírez Ávila *et al.*, 2003a]. The original motivation was to model communication in certain biological systems like fireflies, but it turns out that the oscillator devised to that effect has a wide range of applicability. An LCO is a device made up of a timer chip LM555 in its astable functioning mode, two RC circuits (containing resistors R_λ and R_γ, and a capacitor C), infrared LEDs, and photosensors; the optoelectronic components allow the LCOs to interact with one another. Basically, an LCO is a relaxation oscillator in the sense that it possesses two time scales: within each cycle there are intervals of slow (charging stage) and fast voltage variations (discharging stage, when the LCO fires), and the form of the oscillation is therefore very far from a simple sine wave; it is rather pulse-like. The period of an LCO is linked to the resistors and capacitor values and it is given by $T = T_\lambda + T_\gamma$, where $T_\lambda = (R_\lambda + R_\gamma)C \ln 2$ is the time of the charging process and $T_\gamma = R_\gamma C \ln 2$ that corresponding to the discharging stage. A system of interacting LCOs is an excellent example of pulse-coupled oscillators, just as coupled fireflies that the LCOs mimic. In the system of LCOs, we have understood synchronization as an adjustment of rhythms of self-sustained oscillators giving rise to a phase locking $\Delta\phi_{ij} = \text{const}$[1] and working in conditions when natural disturbances were minimized that permitted us to postulate a model fitting well the experimental results. The presence of perturbations is almost unavoidable, however, it is important to handle them because these fluctuations can substantially modify the system's behavior [García-Ojalvo & Sancho, 1999]. In this paper, we analyze the case in which the oscillators are influenced by a bounded uniform noise in a wide range[2]. The noise acts on the voltage supply (V_M), causing changes to the LCOs' amplitude signal which remains constant for noise-free oscillators. This type of setup could be conceived using a digital noise generator such as that described by Browne [2002]. The choice of uniform noise is related to the fact that for our oscillators it is more natural to have upper and lower voltages within certain limits. On the other hand, it is interesting to work

[1]Even if this definition does not include some nontrivial phenomena

[2]In a forthcoming paper is analyzed the Gaussian noise case

with this kind of noise because it acts strongly on the system. Recently, several works have been devoted to the study of the influence of noise on coupled maps [Kim *et al.*, 2003] or oscillators [Zhou *et al.*, 2002b; Kiss *et al.*, 2004]. Most of them deal with phase synchronization induced by a common noise acting on the system [Teramae & Tanaka, 2004]. Here, we analyze a system of two LCOs under three different configurations: uncoupled, master–slave and mutually coupled, considering noise acting on each LCO. The equations that describe the model for N LCOs are:

$$\begin{aligned}\frac{\mathrm{d}V_i(t)}{\mathrm{d}t} &= \lambda_i[(V_{Mi} + (1-2\zeta_i(t))\sqrt{D}) - V_i(t)]\epsilon_i(t) - \gamma_i V_i(t)[1-\epsilon_i(t)] + \\ &\quad \sum_{i,j}^{N} \beta_{ij}\delta_{ij}[1-\epsilon_j(t)]; \quad i,\ j = 1,\ldots,N. \end{aligned} \tag{1}$$

Here the oscillator state $\epsilon_i(t)$ is given by:

$$\begin{aligned} \epsilon_i(t) &= 1, \quad \text{extinguished LCO} \quad \text{(charge)} \\ \epsilon_i(t) &= 0, \quad \text{fired LCO} \quad \text{(discharge)}, \end{aligned}$$

in which $\epsilon_i(t)$ changes its value when it reaches the upper threshold $(2V_M/3)$ or the lower threshold $(V_M/3)$. The parameters, $\lambda_i = \ln 2/T_{\lambda i}$, $\gamma_i = \ln 2/T_{\gamma i}$ (depending on the LCOs' electronic components) characterize the LCOs' charge and discharge stages, β_{ij} is the coupling strength, $\delta_{ij} = 1$ if the LCOs may interact and $\delta_{ij} = 0$ otherwise. Finally, $\zeta_i(t)$ is a random number chosen from a uniform distribution on the interval $[0, 1]$. The uniform noise is given by $(1 - 2\zeta_i(t))\sqrt{D}$ defining new lower and upper thresholds on a uniform distribution according to the value of $\sqrt{D}$.

We define the instantaneous phase of an LCO (with label i) in accordance with the Poincaré map method [Neiman *et al.*, 1999; Rosenblum *et al.*, 1997; Pikovsky *et al.*, 1997, 2001]:

$$\phi_i(t) = 2\pi\left(\frac{t - t_i^k}{t_i^{k+1} - t_i^k}\right) + 2\pi k$$

This definition of the phase gives the appropriate result in the case of LCOs whether the oscillations are merely periodic or if they are disturbed by noise, since we can consider the beginning of the flashing events as the points lying on the Poincaré section in the phase space. Using this concept, we can define the instantaneous linear phase difference (LPD) between oscillators in terms of the difference among the flashing time of each LCO, considering one of them as the reference oscillator. For a system of two LCOs, the LPD when the LCO_1 achieves its $(k+1)^{\mathrm{th}}$ firing event can be written as:

$$\Delta\Phi_{12}^{\mathrm{linear}}(t_1^{(k+1)}) \;=\; \phi_2(t_1^{(k+1)}) - \phi_1(t_1^{(k+1)}) \;=\; 2\pi \frac{t_2^{(k+1)} - t_1^{(k+1)}}{t_2^{(k+1)} - t_2^{(k)}}, \qquad (2)$$

where $t_1^{(k+1)}$, $t_2^{(k+1)}$ represent the times at which the $(k+1)^{\mathrm{th}}$ firing event of LCO_1 and LCO_2 respectively is produced. Note that the phases $\phi_{1,2}$ are defined on the whole real line and due to the phase slips, it is better to introduce the cyclic relative phase [Schäfer *et al.*, 1999] in order to define phase locking in noisy systems via synchrograms [Schäfer *et al.*, 1998; Neiman *et al.*, 2000]. The cyclic phase difference (CPD) gives a result on the circle $[0:1]$. It is obtained from the LPD as:

$$\Delta\Phi_{12}^{\mathrm{cyclic}}(t_1^{(k+1)}) \;=\; \frac{1}{2\pi}\left[\Delta\Phi_{12}^{\mathrm{linear}}(t_1^{(k+1)}) \mod 2\pi\right]. \qquad (3)$$

Hence, Eqs. (2)–(3) can be used to describe synchronization with stroboscopic observation. Using Eq. (1) we have performed simulations to study the influence of uniform noise on LCOs. In Eq. (1) we associate $\sqrt{D}$ to the noise intensity that we vary in the interval $[0, 4]$.

One way to characterize synchronization in a system of LCOs is computing the statistical moments (mean and variance) for both LPD and CPD. We have found that the mean LPD and the CPD variance exhibit the clearest pictures to detect synchronization, since when it occurs these quantities are almost constant and near to zero. In consequence, we use throughout this paper the variance of the CPD and the CPD probability densities to determine synchronous regimes.

In Sec. 2, we study the uncoupled LCOs case. The results of the case in which an LCO drives another LCO (master–slave) are shown in Sec. 3. The case of two mutually coupled LCOs (same hierarchy) is analyzed in Sec. 4. Finally, we construct an extended state diagram in Sec. 5 showing how the Arnold tongues are modified by the presence of noise in the master–slave and mutual coupling configurations.

2 Uncoupled LCOs

When an LCO is not affected by noise, its period remains constant and we can characterize a system of two uncoupled LCOs by the following facts:

- For identical LCOs ($T_{01} = T_{02}$), the LPD is such that it remains constant but not necessarily zero (black line in Fig. 1(a) and in the inset). Consequently, the CPD probability density, corresponding to the time series, is a δ-function (Fig. 1(b)).

- For non-identical LCOs ($T_{01} \neq T_{02}$), the LPD grows ($T_{01} < T_{02}$) or shrinks ($T_{01} > T_{02}$) with a definite slope as is shown in Fig. 1(f)) (black line). Thus the CPD probability density shows multiple modes (Fig. 1(g)).

Now, considering the noise effects in the systems described above, we note that for the identical LCOs system, the LPD may behave in a diffusive way, performing a motion that recalls a random walk (red line in the zoomed region of Fig. 1(a)). When the noise is quite strong, the picture that is obtained reminds turbulence (green line in Fig. 1(a)). The CPD probability densities for different noise intensities are depicted in Figs. 1(b)–(e).

If the LCOs are not identical, weak noise intensities do not change significatively the LPD evolution even for large times. When the noise intensity is sufficiently strong, the LPD evolution at the beginning remains almost constant as in the case of weak noise intensities but as time goes on, the noise effects become significant as shown in Fig. 1(f) (red line). Even though the LPD still decreases, a random walk-like behavior can be seen along the corresponding straight slope. For strong noise intensities, the LPD behavior is similar to that observed in the corresponding case of identical LCOs, i.e. in a turbulent-like way. It is interesting to note that when the noise is strong, both CPD probability densities (identical and non-identical LCOs) are quite similar suggesting that the intrinsic differences between the systems are not relevant when noise intensity is very strong. In this case, the LCOs' periods can change abruptly and they are far from being constant.

For the probability densities, it is better to work with the normalized (cyclic) phase difference, which characterizes the system in a better way and avoids a confusing interpretation. For instance, in the case of very strong noise, the LPD distribution shows a unimodal picture, but this does not mean that a definite value of LPD is more probable; rather it is only the effect of the LPD diffusing and decaying in time. On the contrary, CPD distributions show the stochastic behavior of the system and remain quite uniform even for strong noise. Therefore, the CPD probability density will be one of our main tools to analyze LCO systems.

3 Master–Slave Configuration

We now consider the case where one LCO (the master) drives another (the slave), in the sense that the master influences the slave but is blind to the light coming from the slave and from the environment. We expect that synchronous behavior is present when the LCOs are not very different and when they are noise-free. Let us take LCO_2 as the master-LCO.

When the LCOs are identical (Fig. 2(a)), we observe that the synchronization phenomenon is only produced when the noise intensity vanishes (Fig. 2(b)), i.e. any noise influencing the system destroys the synchronous regime (Figs. 2(c)–(d)). Figure 2(a) is explained by the fact that noise shrinks the synchronization region (see the Arnold tongues in Fig. 4(a) of Sec. 5) and as a consequence, for the case of identical oscillators, the synchronous regime is lost even with a small noise intensity. This may be explained due to the dynamics of the master–slave configuration since the impulses of the master modify the slave's period (most often shortening it) and since the synchronous regime is weakly stable for the case of identical LCOs under a master–slave configuration, the action of noise produces desynchronization between the LCOs. On the other hand, when the LCOs are different (Fig. 2(e)), the synchronous region is maintained when the noise intensity is weak (Figs. 2(f) and (i)). In Fig. 2(e), we can see that the synchronous regime "resists" better to noise influence when the master-LCO is more different from the slave LCO, i.e. the greater the difference among LCOs, the greater the synchronous region, but the transition region becomes larger too. It is clear that this last point is valid only when the LCOs' characteristics are such that the values are inside an Arnold tongue (see [Ramírez Ávila *et al.*, 2003b] and Sec. 5); otherwise, we cannot refer to synchronization. In this case, due to the fact that the master-LCO has a period less than the slave-LCO, noise acts to stabilize the system. The CPD probability densities are shown to indicate the transition from synchronization to desynchronization. Finally, it is interesting to note the shape of the curves in Figs. 2(a) and (e): the CPD variance exhibits a behavior similar to a phase transition and it tends to a constant value ≈ 0.083 when the system does not synchronize, which corresponds to a uniform CPD distribution.

4 Mutual Interaction

We now consider the case when the LCOs have the same hierarchy, considering symmetrical coupling, i.e. $\beta_{12} = \beta_{21}$ and again we analyze a system with identical LCOs (Figs. 3(a)–(i)), and with non-identical LCOs (Figs. 3(j)–(r)). For identical LCOs, we observe a large transition region (Fig. 3(a)) in which for small noise intensity values ($\sqrt{D} = 0.2$, $\sqrt{D} = 0.4$), the LPD evolution remains around zero (Figs. 3(b)–(c)), indicating that synchronization is still present, confirmed by the CPD probability densities which contain peaks only in the extremal values of the histogram (Figs. 3(f)–(g)). For moderate values ($\sqrt{D} = 1.75$), the LPD evolution presents some phase slips so that there are epochs where the LPD oscillates around multiples of 2π, with dynamics similar to a random walk behavior (Fig. 3(d)) and the CPD probability density shows a kind of transition process (Fig. 3(h)). On the other hand, for large noise intensity values ($\sqrt{D} = 2.5$), the LPD evolution is random walk-like (Fig. 3(e)) and the CPD probability density is broad (Fig. 3(i)), indicating that synchronization is lost.

In the case in which the LCOs are not identical, the values used for the LCO_2 period are $T_{02} = 33.5$ ms (cyan line), and $T_{02} = 34.5$ ms (magenta line), which are symmetric with respect to $T_{01} = 34.0$ ms. The CPD variance curves as a function of noise intensity are almost identical with a slight shift in the synchronous region (Fig. 3(j)). For small noise intensity values ($\sqrt{D} = 0.2$), synchronization is present for both cases (Figs. 3(k) and (o)). At the maxima of the transition regions ($\sqrt{D} = 0.4$), several phase slips appear, especially when $T_{02} < T_{01}$ (Fig. 3(l)) and the CPD probability density again reveals a transition process (Fig. 3(p)). For noise intensity values situated in the desynchronization region ($\sqrt{D} = 1.75$ and $\sqrt{D} = 2.5$), the LPD evolution decays or grows (Figs. 3(m)–(n)) like in the case of uncoupled LCOs, suggesting that noise disturbance acts in a way that the coupling is not important. Consequently, the CPD probability densities for both cases are broad (Figs. 3(q)–(r)). We remark again that the CPD variance is ≈ 0.083 when the system is not synchronized.

5 Arnold Tongues

Many phenomena occurring in two coupled oscillators can be studied by means of Arnold tongues [Arnol'd, 1965] which are useful tools to describe phase-locking, quasi-periodic or chaotic behavior and the transitions between these states [Argyris *et al.*, 1994; Nicolis, 1995]. In order to characterize the synchronous regions in coupled oscillators of different nature, it is useful to build the Arnold tongues [Glass & Perez, 1982; Matsumoto *et al.*, 1987; Glass & Sun, 1994; Coombes & Bressloff, 1999; Yoshino *et al.*, 1999; Stoop *et al.*, 2000; Ramírez Ávila *et al.*, 2003b] that are defined as the state diagram (generally on the frequency detuning, coupling strength plane) in which all regions of synchronization have the form of vertical tongues [Pikovsky *et al.*, 2001], i.e. the motion is periodic inside these regions. As expected, the area of Arnold tongues diminishes with noise intensity. In the case of the master–slave configuration, the boundary values' shift is quite asymmetric; for small noise intensities, in practice, there is no shift in the left-boundary value. On the other hand, the right-boundary value shift is significant. We can observe left-boundary shifts with high noise intensities but the right-boundary shift becomes very large as well, i.e. the asymmetric behavior on the boundary values shift is still present, as shown in Fig. 4(a). Choosing the tongue related to $\sqrt{D} = 0.5$ and considering $\beta = 166$, we can see that for $T_{02} = 32.7$ ms, i.e. a value slightly below the left-boundary, the LPD evolution suffers several phase slips (Fig. 4(b)) and it is expected to fall steadily; consequently, the CPD probability density is not a δ-function and shows a tendency towards a uniform distribution (Fig. 4(c)). When $T_{02} = 32.8$ ms, i.e for the left-boundary, we see that the LPD remains constant (Fig. 4(d)) and the CPD probability density tends to a δ-function (Fig. 4(e)). When $T_{02} = 32.9$ ms, i.e. a value inside to the Arnold tongue, the LPD very quickly attains the synchronous state and it is levelled out in time (Fig. 4(f)), and as a result, the CPD probability density can be considered a δ-function (Fig. 4(g)).

In Fig. 4(h), the Arnold tongues are shown for the mutual coupling configuration. The tongues structure suggests that there exists certain symmetry with respect to the fixed value of LCO_1 period ($T_{01} = 34.0$ ms), especially when the noise intensity is small or null and void, and the coupling strength is not very strong (e.g. for zero noise intensity and $\beta = 166$, the left and right boundaries are $T_{02} = 33.4$ ms, and $T_{02} = 34.6$ ms respectively, showing that there is symmetry with respect to $T_{01} = 34.0$). On the other hand, for zero noise intensity and $\beta = 500$ the boundaries are $T_{02} = 32.25$ ms, and

$T_{02} = 36.00$ ms, showing that there is not perfect symmetry with respect to $T_{01} = 34.0$ ms. As in the master–slave case, the tongues become smaller when the noise intensity increases and their corresponding LPD evolution and CPD probability density graphs show exactly the same behavior as in the master–slave case. Choosing again the tongue related to $\sqrt{D} = 0.5$ and considering $\beta = 166$, we see the same behavior as in the preceding case: near to the right boundary (Figs. 4(i)–(j)), on the right boundary (Figs. 4(k)–(l)), and inside the tongue (Figs. 4(m)–(n)). If we represent the same graphs with the same reference LCO period but for cases with different noise intensities, the behavior could be completely different. To conclude, the Arnold tongues show that in the master–slave configuration, the LCOs can be very different. However, the condition that the master's period must be strictly equal to or less than the slave's period is necessary in order to achieve synchronization. In the mutual coupling case, however, the periods may be different. Nevertheless, in comparison with the master–slave case, the LCOs periods must not be very different.

6 Conclusions

We have observed, as expected from statistical mechanics, that uniform noise induces disordering phase transitions, i.e. the higher the intensity of fluctuations, the larger the disorder. For identical uncoupled LCOs, the noise makes the LPD evolve in a random walk-like way or in a turbulent-like way. For non-identical and uncoupled LCOs, the LPD evolution tends to decay or to grow depending on the periods of the LCOs. For non-identical and uncoupled LCOs, the LPD evolution tends to decay or to grow depending on the periods of the LCOs. We expect that two uncoupled LCOs subjected to a common correlated noise could exhibit noise-induced transitions similar to those described by Zhou & Kurths [2002a]. In the master–slave configuration, even very weak noise intensities destroy the synchronous state when the LCOs are identical. This allows us to conclude that for this configuration the LCOs' periods have to be different but contained inside the Arnold tongues in order to maintain the synchronization and compete against the disturbances. When noise acts on identical mutually coupled LCOs, the transition region is quite large, which could indicate that the system exhibits a certain robustness to the noise, in the sense that it is possible to observe synchronization in a statistical sense despite the action of noise with larger intensity values. On the contrary, in the case of non-identical LCOs, the transition region is narrow and the system exhibits a broad distribution (desynchronization) for noise intensity values significantly smaller than in the previous case. As stated by several authors [Stratonovich, 1963; Heagy *et al.*, 1995], noise can induce phase slips in periodic oscillators; we found a similar behavior in which the synchronization-desynchronization transition is characterized by more frequent phase slips until the LPD decays or grows like in the uncoupled case as the system is more affected by noise. It is clear that very high noise intensity values act in a way that coupling seems to be broken. Concerning the phase slips, their number is in a close relationship with the noise intensity: the higher the noise intensity, the greater the number of phase slips. In all the cases, the CPD variance is a good indicator to identify the system as being in its synchronous or asynchronous state; when there is no synchrony, the CPD variance takes a value approximatively equal to 0.083. The CPD probability density can exhibit the passage from synchronization to desynchronization with a shape typical of a phase transition-like phenomenon.

Acknowledgements

We are indebted to F. Moss whose ideas and interesting discussions were the source of inspiration of this paper. We thank G. Nicolis for his valuable comments. We are grateful to D. Sanders for careful reading of the manuscript. JLD is research associate of the Belgian National Foundation for Scientific Research.

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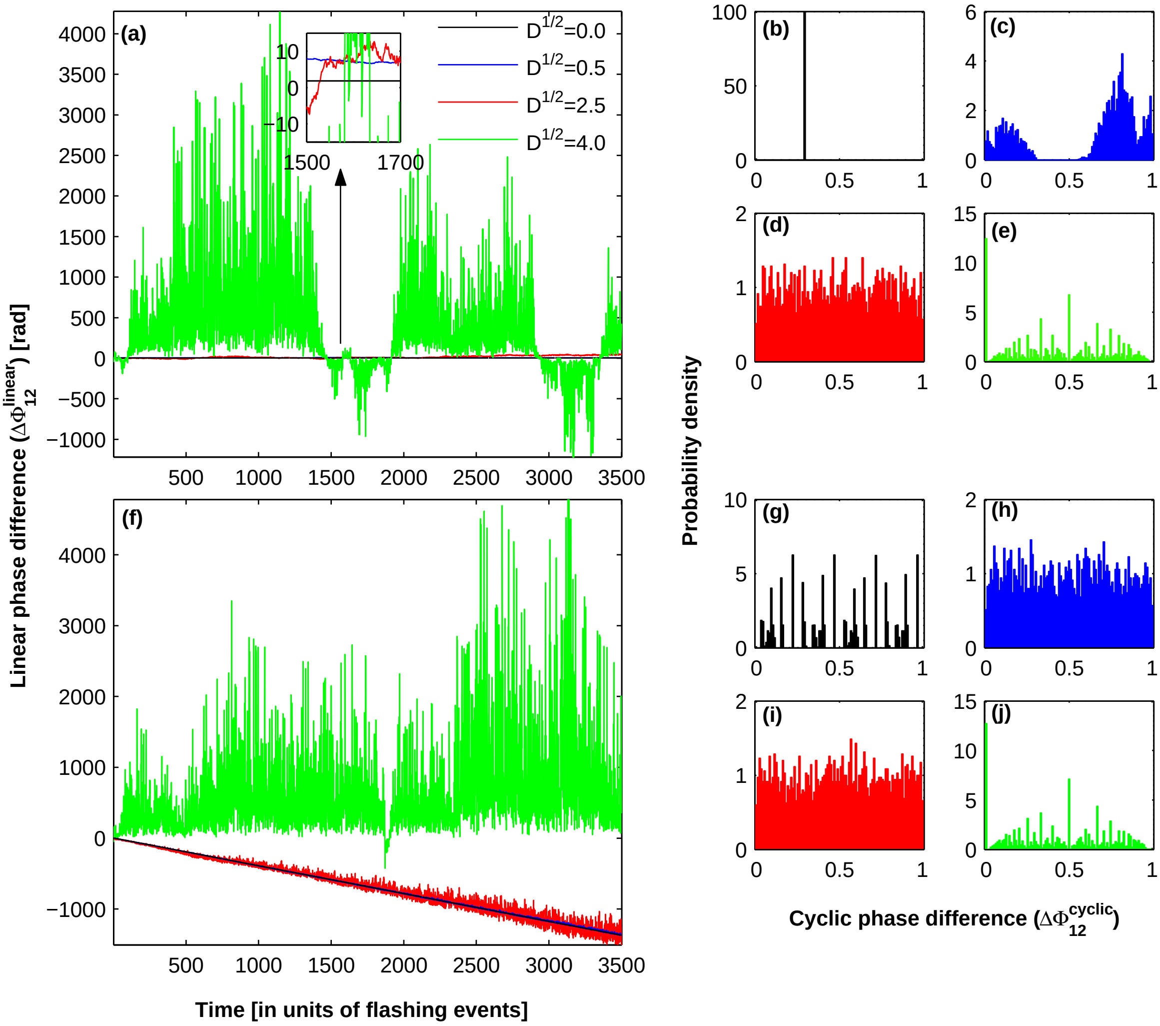

Figure 1: Period values of a system of two uncoupled LCOs: $T_{01} = T_{02} = 34.0$ ms ((a)–(e)), and $T_{01} = 34.0$ ms, $T_{02} = 32.0$ ms ((f)–(j)). LPD evolutions and the CPD probability densities for different noise intensities. (a) Identical LCOs LPD evolution. For $\sqrt{D} = 0.0$, the LPD is constant (black line). The inset is a magnification in the region ranging from 1500 to 1700 time units (flashing events), where the LPD evolution is clearer. (b)–(d), the corresponding probability densities when $\sqrt{D} = 0.0$, $\sqrt{D} = 0.5$, $\sqrt{D} = 2.5$, and $\sqrt{D} = 4.0$ respectively. (f) Non-identical LCOs LPD evolution. (g)–(j), the corresponding CPD probability densities as above. The evolution and the statistics is taken over $N_T = 3500$ flashing events.

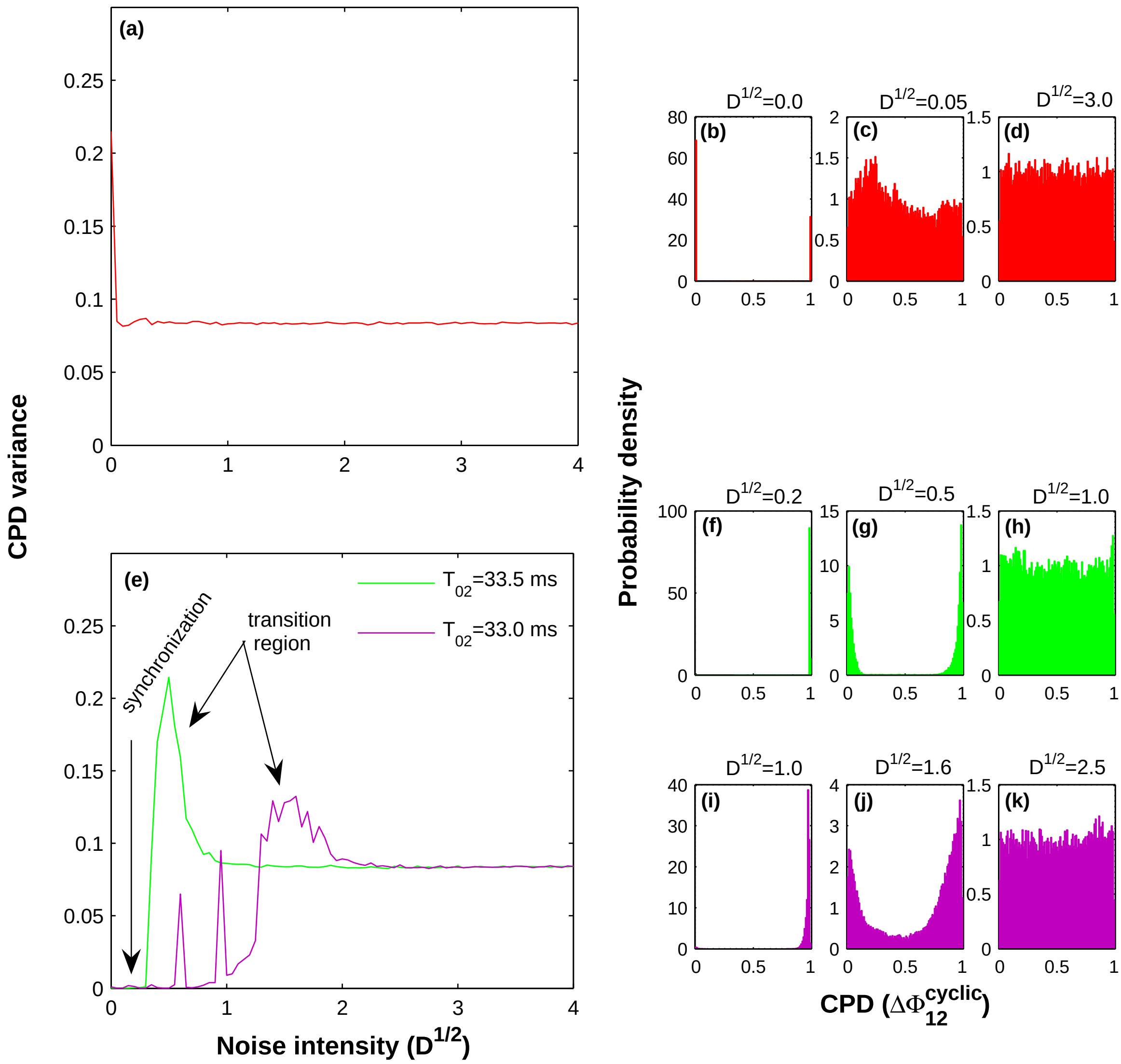

Figure 2: CPD variance as a function of noise intensity and CPD probability densities for different noise intensities when the configuration is master–slave. The period value for the slave LCO is $T_{01} = 34.0$ ms and the coupling strength $\beta = 166$. (a)–(d) Identical LCOs case. (e)–(k) Non-identical LCOs case. The period values used for LCO_2 are $T_{02} = 33.5$ ms (green line), and $T_{02} = 33.0$ ms (magenta line). The statistics were taken over 30000 flashing events.

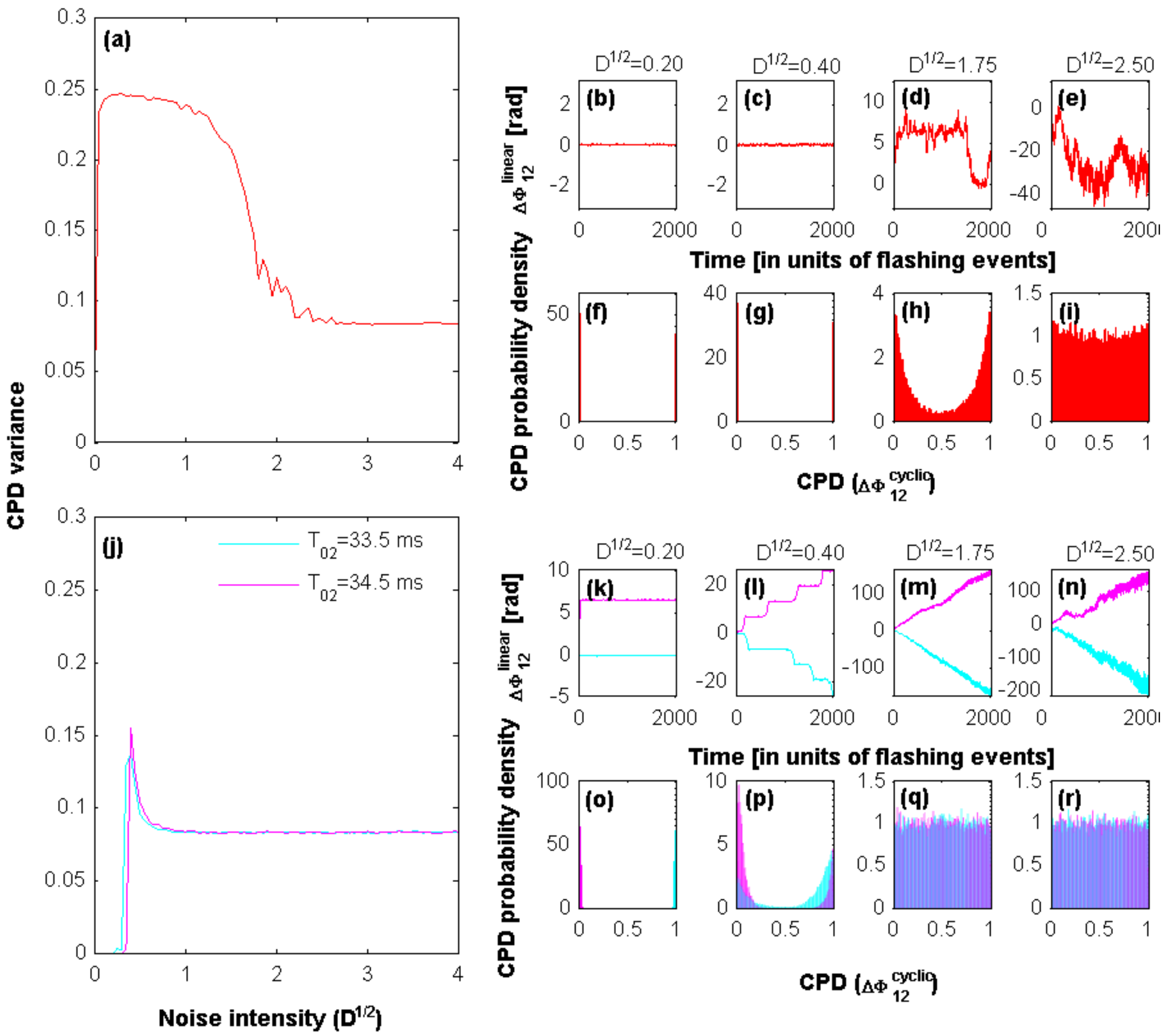

Figure 3: CPD variance as a function of noise intensity for mutually interacting LCOs. (a) Identical LCOs case. (b) Non-identical LCOs case, where the period values used for LCO_2 are $T_{02} = 33.5$ ms (cyan line), and $T_{02} = 34.5$ ms (magenta line). The parameter values for T_{01} and β are the same that in Fig. 2. The LPD evolution and CPD probability densities for different noise intensities are shown to indicate the transition from synchronization to desynchronization for (b)–(i) identical LCOs and (k)–(r) non-identical LCOs. As in the master–slave case, the statistics were made over 30000 flashing events.

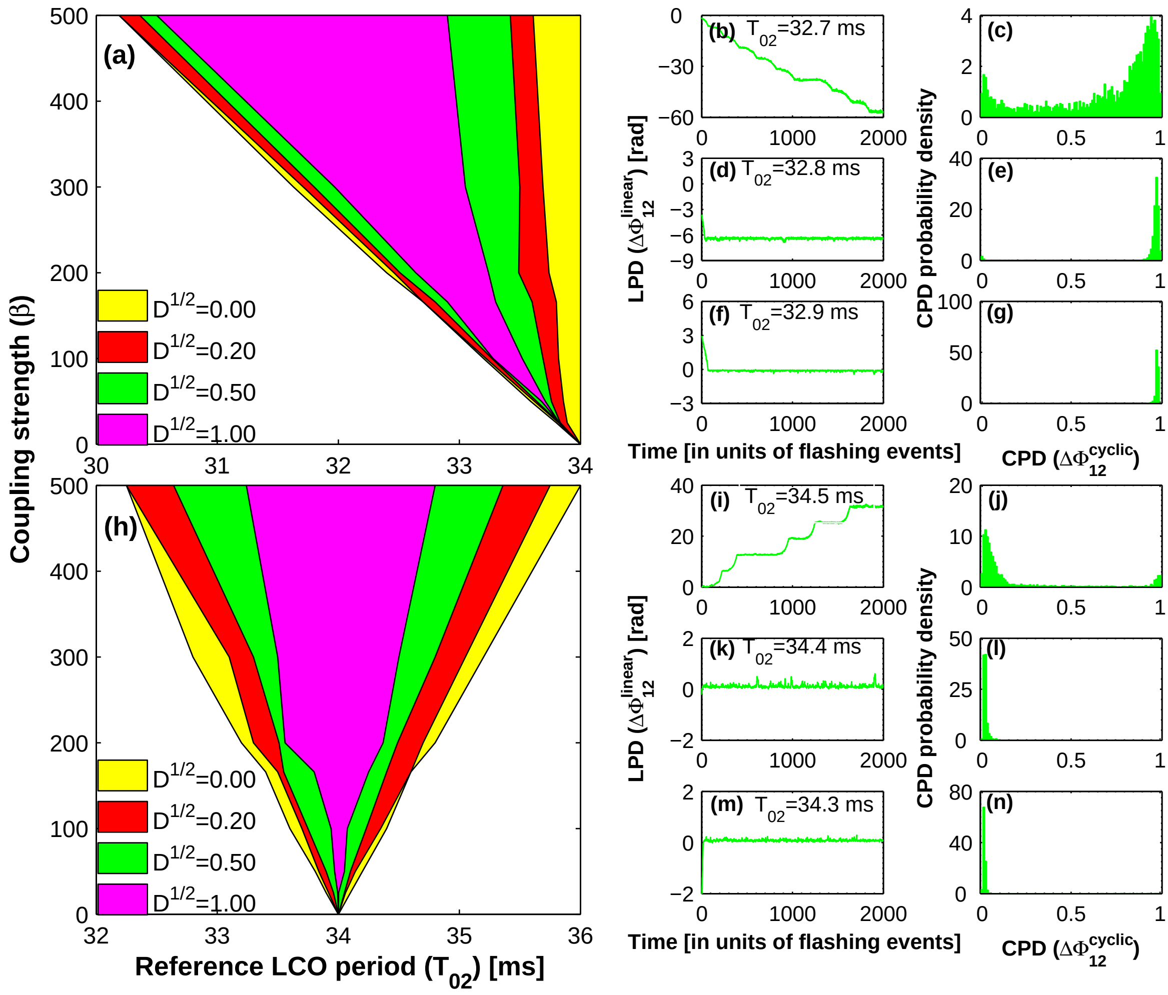

Figure 4: (a) Arnold tongues for a master–slave system. The LPD evolution and CPD probability density for period values (b)–(c) slightly left-outside (T_{02} = 32.7 ms), (d)–(e) on the left-boundary (T_{02} = 32.8 ms), and (f)–(g) inside (T_{02} = 32.9 ms) the Arnold tongue (j) Arnold tongues for mutually coupled LCOs. The LPD evolution and CPD probability density for period values (i)–(j) slightly right-outside (T_{02} = 34.5 ms), (k)–(l) on the right-boundary (T_{02} = 34.4 ms), and (m)–(n) inside (T_{02} = 34.3 ms) the Arnold tongue. For both cases, we have considered the tongue corresponding to the system perturbed with noise intensity $\sqrt{D} = 0.50$ (green patch) and the coupling strength $\beta = 166$.