

Synchronization in chains of light-controlled oscillators

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Abstract. Using light-controlled oscillators (LCOs) and a mathematical model of them introduced in [1], we have analyzed a population of LCOs arranged in chains with non-periodic (linear configuration) and periodic (ring configuration) boundary conditions in which we have solved numerically the corresponding equations for a broad interval of coupling strength values and for chains between 2 and 25 LCOs. We have considered three different situations, viz. identical LCOs, identical LCOs with simplifications (LCOs considered as integrate-and-fire (IF) oscillators), and finally nonidentical LCOs. We study synchronization under two criteria: the first takes into account the simultaneity of flashing events (phase difference criterion), and the second considers period-locking as a criterion for synchronization. For each case, we have identified regions of synchronization in the plane coupling strength versus number of oscillators. We observe different behaviors depending on the values of these variables.

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1. Introduction

Several works have been devoted to the study of locally coupled oscillators in one-dimensional configurations and considering various kinds of oscillators. Phenomena like phase-locking [2, 3], sequences of oscillators with the same frequencies [4], phase transitions [5], pattern formation [6], travelling waves [7], and clustering [8] are common in chains of oscillators; all these phenomena are linked to synchronization, i.e. the adjustment of rhythms of oscillating objects due to their interaction [9]. Integrate-and-fire (IF) oscillators have been used extensively to describe and model a great variety of phenomena such as synchronization in fireflies [10]. An IF oscillator is a typical relaxation oscillator and is characterized by a voltage-like state variable V which grows until it reaches a threshold then the oscillator “fires” and V is reset instantaneously to its baseline. Recently, a new class of oscillators describing synchronization in a very easy experimental way has been introduced [1]. The original motivation was to model communication in certain biological systems like fireflies, but it turned out that the oscillator devised to that effect has a wide range of applicability. This work is an extension of previous results obtained with small chains of 2 and 3 LCOs. Here we extend to a large number of LCOs (up to 25) and with two close configurations. In this paper, we study and characterize the synchronous behavior in linear and ring configurations of LCOs. In section 2 we describe the model and the

synchronization criteria. In section 3 we present our results. Finally, in section 4 we discuss the results and compare them with results obtained in related works.

2. The model

As described in [11, 1], an LCO is an electronic relaxation oscillator in the sense that it possesses two time scales: within each cycle there are intervals of slow (charging stage) and fast voltage variations (discharging stage, during which the LCO fires). Each LCO consists of an LM555 chip wired to function in its astable oscillating mode (figure 1). The beginning of the charging and the discharging stage are determined by two well-defined thresholds at $V_M/3$ and $2V_M/3$ respectively; V_M being the voltage source. The period of an LCO is related to the resistors (R_λ, R_γ) and capacitor (C) values and is given by $T = T_\lambda + T_\gamma$ where $T_\lambda = (R_\lambda + R_\gamma)C \ln 2 = \ln 2/\lambda$ is the time interval in which the charging process takes place and $T_\gamma = R_\gamma C \ln 2 = \ln 2/\gamma$ that corresponding to the discharging stage. An LCO is characterized by the output voltage $V(t)$ taken in the pin 3 of the LM555 chip. The LCOs may interact between them by means of light pulses it is why they are equipped with photo-sensors and LEDs that allow the optical coupling (figure 1). The photo-sensors act as current sources when they are receiving light, shortening the charging time of the capacitor and making longer the time required to discharge.

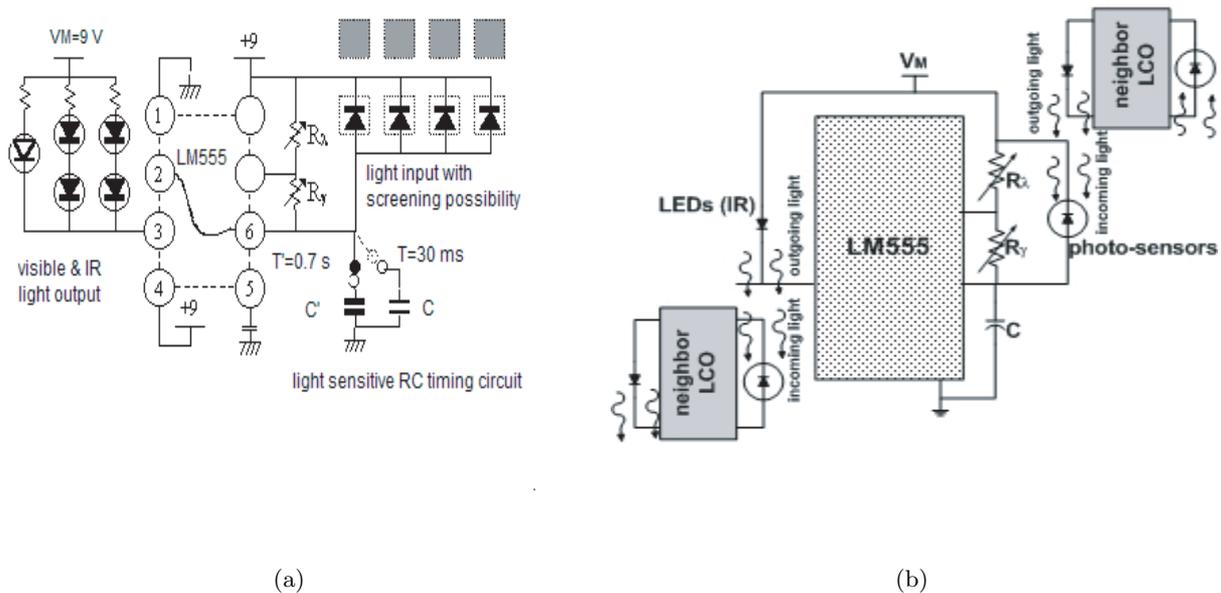


Figure 1. (a) Block diagram of an LCO. (b) Simplified diagram of the LCO and schematic view of the coupling between LCOs.

The equations that describe the model for N LCOs are:

$$\frac{dV_i(t)}{dt} = \lambda_i[(V_{Mi} - V_i(t))\epsilon_i(t) - \gamma_i V_i(t)[1 - \epsilon_i(t)] + \sum_{i,j} \beta_{ij} \delta_{ij} [1 - \epsilon_j(t)], \quad i, j = 1, \dots, N, \quad (1)$$

where β_{ij} is the coupling strength, $\delta_{ij} = 1$ if the LCOs may interact and $\delta_{ij} = 0$ otherwise, and $\epsilon_i(t)$ is the oscillator state that takes the value 1 (charging stage) or 0 (discharging stage); $\epsilon_i(t)$

changes its value when it achieves the upper threshold ($2V_M/3$) or the lower threshold ($V_M/3$). We must mention that the model has been validated experimentally [1].

Using (1), we can write the equations corresponding to each one of the studied cases. In the case of identical LCOs in a linear configuration:

$$\frac{dV_1(t)}{dt} = \lambda(V_M - V_1(t))\epsilon_1(t) - \gamma V_1(t)[1 - \epsilon_1(t)] + \beta[1 - \epsilon_2(t)] , \quad (2)$$

$$\frac{dV_i(t)}{dt} = \lambda(V_M - V_i(t))\epsilon_i(t) - \gamma V_i(t)[1 - \epsilon_i(t)] + \beta[2 - \epsilon_{i-1}(t) - \epsilon_{i+1}(t)] , \quad 1 < i < N , \quad (3)$$

$$\frac{dV_N(t)}{dt} = \lambda(V_M - V_N(t))\epsilon_N(t) - \gamma V_N(t)[1 - \epsilon_N(t)] + \beta[1 - \epsilon_{N-1}(t)] . \quad (4)$$

For identical LCOs in a linear configuration neglecting the influence on the discharge, the coupling strength β acts only on the charging term. As a consequence, β will always be attached to the charging term, and the time in which the discharging process (flash) takes place is constant. The equations in this case are given by:

$$\frac{dV_1(t)}{dt} = [\lambda(V_M - V_1(t)) + \beta(1 - \epsilon_2(t))]\epsilon_1(t) - \gamma V_1(t)[1 - \epsilon_1(t)] , \quad (5)$$

$$\frac{dV_i(t)}{dt} = [\lambda(V_M - V_i(t)) + \beta(2 - \epsilon_{i-1}(t) - \epsilon_{i+1}(t))]\epsilon_i(t) - \gamma V_i(t)[1 - \epsilon_i(t)] , \quad 1 < i < N \quad (6)$$

$$\frac{dV_N(t)}{dt} = [\lambda(V_M - V_N(t)) + \beta(1 - \epsilon_{N-1}(t))]\epsilon_N(t) - \gamma V_N(t)[1 - \epsilon_N(t)] . \quad (7)$$

For nonidentical LCOs in a linear configuration, the description is made with (2)–(4) except for the fact that each LCO has a different parameter λ . For the ring configuration, the above equations remain essentially unchanged with the condition $\epsilon_{N+1} = \epsilon_1$ due to the periodic boundary conditions.

3. Results

Using experimental and numerical observations, we showed in [1] a strong dependence on initial conditions even for a system composed of only 2 LCOs. We solved the equations numerically for chains of between 2 and 25 LCOs, varying the coupling strength from $\beta = 10$ to $\beta = 1000$ in steps of $\Delta\beta = 10$ and performing 100 simulations for each case to improve the statistics. We use the following parameter values: $R_\lambda = 100$ k Ω , $R_\gamma = 1.6$ k Ω , $C = 0.47$ μ F. When the LCOs are nonidentical, $\lambda_i = 1/(R_\lambda + \xi_i + R_\gamma)C$, where ξ_i is a random number following a Gaussian distribution with mean equal to zero and variance equal to $10^4 \Omega$. In the case in which synchronization is not achieved, we observe a minimum of 7500 flashing events.

We use two approaches to study synchronization: the first considers almost simultaneous firing events with constant phase differences, and the second considers equality of periods as the criterion for synchronization. The phase difference criterion (PDC) is very strong, in the sense that we only consider as synchronized those LCOs flashing almost simultaneously and keeping their phase difference constant, whereas in the period criterion (PC), simultaneity of flashing events is not strictly necessary. In order to visualize the synchronous behavior of the LCOs using both criteria we construct figures 2 and 3 which show the projection onto the plane of the number of LCOs and (N)-coupling strength (β), with the percentage of synchronization events (PSE) represented by the color scale.

We find that the surfaces generated by the above criteria and configurations are not simply correlated. In order to determine which configuration maximizes the PSE, we obtain the mean value and the standard deviation (for all N and for all β) of the PSE (see table 1). Using the PC, the PSE values are greater than using the PDC, since as we stated before, the latter is a stronger

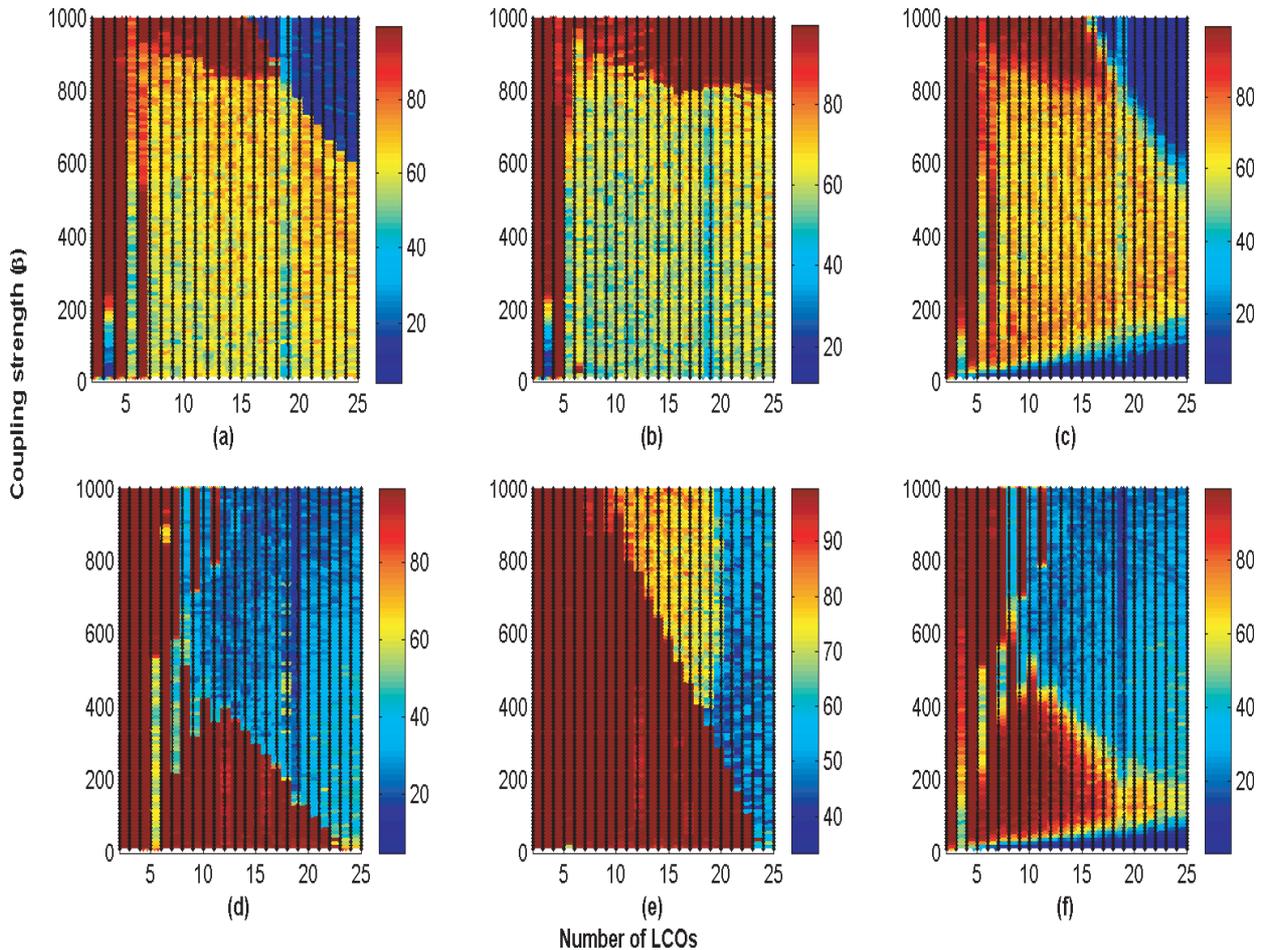


Figure 2. Phase difference criterion for synchronization in linear (first row) and ring (second row) configuration for identical LCOs (a) and (d), identical LCOs neglecting the changes on the discharging stage (b) and (e), and nonidentical LCOs (c) and (f). The color represents the percentage of synchronized events

criterion. From the results shown in table 1 we also note that considering rings of identical LCOs for which we neglect the action on the discharging stage, the PSE is greater than in the other cases, for both PDC and PC. This is not surprising in the sense that neglecting effects on the discharge, the LCOs approach integrate-and-fire oscillators for which total synchronization is attained [10], i.e., 100% in terms of PSE. Now, if we compare the situations for identical and nonidentical LCOs, we note that using PDC, the value of PSE is slightly greater for identical LCOs than using the PC; on the contrary, using the PC the result reverses and PSE values are slightly greater in the case of nonidentical LCOs. This may signify that coupled nonidentical LCOs synchronize forming clusters with equal period but flashing at different times.

4. Discussion

Our results show that there is no exactly predictable behavior. This may be related to the fact that the LCOs are sensitive to initial conditions, as we have demonstrated for 2 and 3 LCOs [1]. Thus, the larger the population, the more difficult it is to predict the behavior of the system. Nevertheless, the simulations allow us to identify regions where it is more probable to

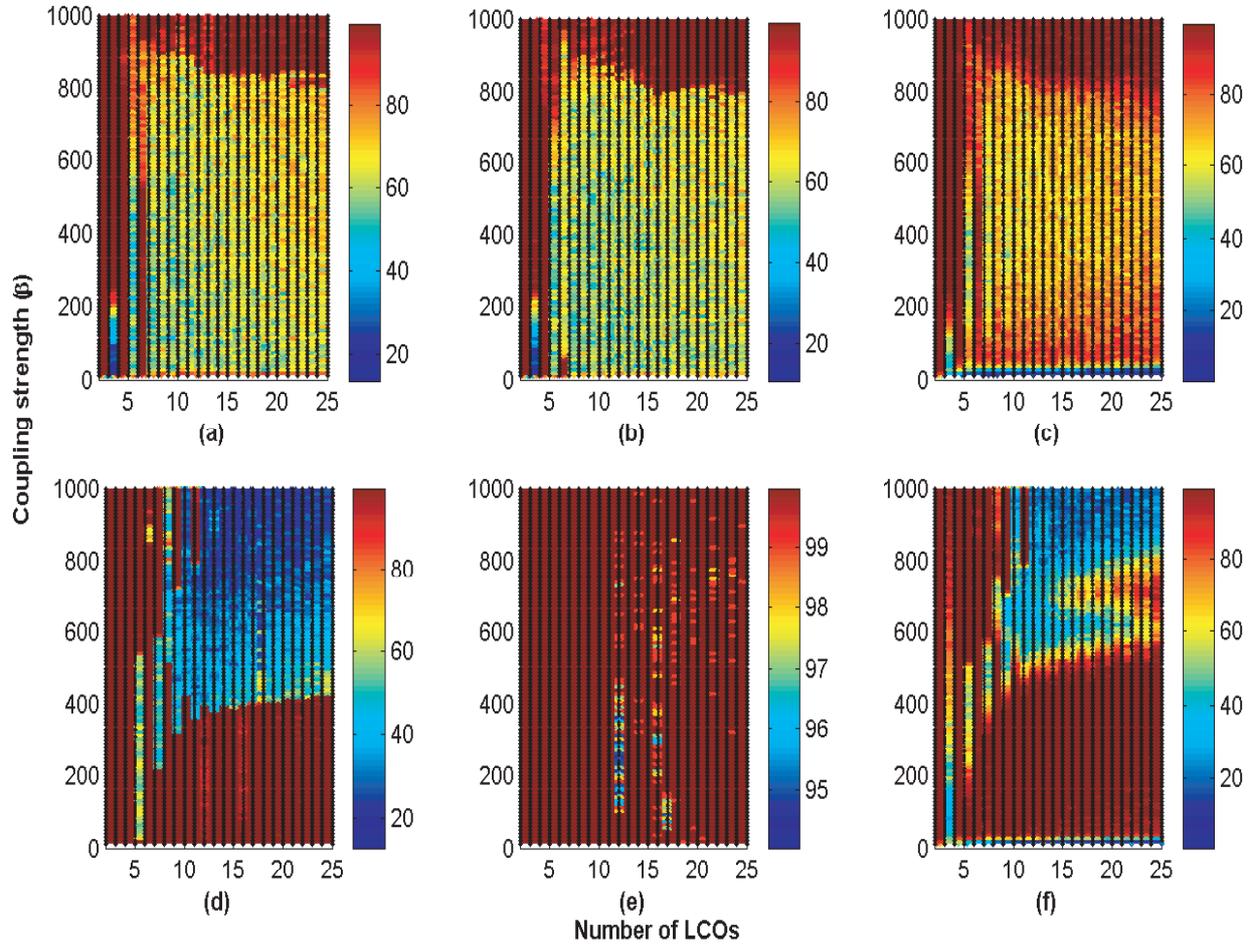


Figure 3. Same as in figure 2 but using the period criterion for synchronization.

Table 1. Mean values and standard deviations of PSE, using all values of N and β for LCOs coupled in linear and ring configuration and using as criterion the boundedness and constancy of the phase difference and the equality of the periods respectively.

Configuration	Synchronization criterion	
	phase difference	period
linear		
identical LCOs	66.46 ± 7.33	73.55 ± 4.69
identical LCOs neglecting action on the discharge	70.45 ± 5.03	71.14 ± 4.84
nonidentical LCOs	64.06 ± 7.40	77.81 ± 3.23
ring		
identical LCOs	58.41 ± 12.59	69.32 ± 12.20
identical LCOs neglecting action on the discharge	85.40 ± 6.86	99.88 ± 0.43
nonidentical LCOs	56.99 ± 9.50	77.75 ± 9.64

observe total synchronization. The results show that in a linear chain, total synchronization is common for a small number of LCOs but for 3 and 5 LCOs the PSE plummets dramatically;

for more than 6 LCOs, the PSE is around 70% in a broad coupling strength region. Neglecting the action on the discharging stage, for identical coupled LCOs in a linear chain configuration, both synchronization criteria give nearly identical results. The difference with respect to the full-model case is that for strong coupling the PSE achieves values near of 100% using the simplified model, a situation that it is not present in the full-model where using the phase difference as a synchronization criterion the PSE drops drastically for strong coupling. In the case of linear chains of nonidentical LCOs, depending on the synchronization criterion, the β vs. N plane exhibits regions that differ considerably from the identical LCOs case. For instance, in the case of weak coupling a domain with low PSE appears using the PDC; however, in the remainder of the plane, the PSE is slightly greater. For ring chains and for weak coupling, both criteria show PSE near to 100% which does not seem to depend on the number of LCOs; nevertheless, for 5 LCOs that is not the case and this can be related to the results found for 5 oscillators in chains of multiple-coupled oscillators in [3], where the oscillators at the ends “drift” away and are no longer able to phase lock with the interior oscillators. For a so called “double ring” configuration by [12] (a ring chain for us) with nonidentical oscillators, they found that synchronous state collapses for a 13 oscillators geometry, having as a consequence that synchronization for large N is not sustained but produces a form of weak synchronization in the chain, i.e., groups of neighboring oscillators run at the same average frequency, but not necessarily with a fixed phase relationship. We have obtained a similar result to [12] but for both cases, viz. identical and nonidentical LCOs in a ring configuration for a broad coupling region. Moreover, these results confirm the correctness of using both synchronization criteria. Neglecting the influence on the discharging stage for identical coupled LCOs in a ring chain, we found that the PSE is practically 100% in all the cases, as in the results found by several authors (see e.g. [13, 14]). They claim that one-dimensional and two-dimensional lattices of IF oscillators always synchronize even this takes a long time. The synchronization time is around 100 periods for a 40×40 lattice according to [13] or the indicated by [14], who claim that the synchronization time average scales logarithmically with the system size. Obviously, neglecting the effects on the discharging time constitutes an approximation to an IF oscillator. We have found that on average, the situation of nonidentical LCOs exhibits slightly greater values of PSE than in the identical case, so that, a deeper insight into situations in which LCOs are nonidentical could be important to confirm this difference. Our hypothesis is that it could be confirmed, since real systems that achieve robust synchrony are composed of nonidentical oscillators and small differences could enhance synchronization as observed in nature [15].

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